Question 1

1. $A \cup B = \{1, \beta, 20, \text{two}\}$
2. $A \cup B = \{\beta, 20\}$
3. $C \setminus A = \{\text{one}, \alpha\}$
4. $\{1, \beta, 20\}$

Question 2

1. $\{x : -10 < x < 3\}$
2. $\{2n : n \in \int\}$
3. $\{n^2 : n \in \mathbb{N}\}$

Question 3

Checking the base case $n = 1$ is trivial. Then for the inductive step, assume for $m \in \mathbb{N}$

$$\sum_{k=1}^{m} q^k = \frac{q^{m+1} - q}{q - 1}$$

Then for $m + 1$,

$$\sum_{k=1}^{m+1} q^k = \sum_{k=1}^{m} q^k + q^{m+1} = \frac{q^{m+1} - q}{q - 1} + q^{m+1}$$
$$= \frac{q^{m+1} - q}{q - 1} + \frac{q^{m+2} - q^{m+1}}{q - 1} = \frac{q^{m+2} - q}{q - 1}$$

therefore, by induction, we complete our proof.

Question 4

Checking the base case $n = 1$ is trivial. Then for the inductive step, assume for $m \in \mathbb{N}$

$$5^m + 5 < 5^{m+1}$$

This implies

$$5^{m+2} > 5 \cdot 5^{m+1} > 5(5^m + 5) > 5^{m+1} + 25 > 5^{m+1} + 5$$

hence by induction, we complete our proof.

Question 5

Let $T = \{2n+1 : n \in \mathbb{N}\}$, since $T \subseteq \mathbb{N}$, T is also well ordered. Then the proof follows by contradiction. Assume there exists a set $S = \{P(n) : n \in T\}$ such that there exists $P(m)$ that is false for $m \in T$. Therefore, there exists $F = \{P(n) : P(n) = \text{False}\} \neq \varnothing$. Since S is well-ordered (since T can be used to index S) and $F \subseteq S$, F must have least element $P(l)$. Since $l \neq 1$, by base case, there exists $l - 2 \in T$. Since $P(l-2) \notin F$, $P(l-2)$ is true, which implies $P(l)$ is true by our inductive step assumption. Therefore, by contradiction, every $P(n) \in S$ must be true.